COMPOSITE FUNCTIONS AND THEIR DOMAINS

A composite function is one in which the output of one function becomes the input for another. The notation is (fDg)(x) or f(g(x)), read "f of g of x", where f and g are both functions of x. In f(g(x)), the g(x) function is substituted for x in the f(x) function. Similarly, in g(f(x)), the f(x) function is substituted for x in the g(x) function. Examples: 1. If f(x) = 2x + 1 and g(x) = x, then (fg)(x) or f(g(x)) = 2D x + 1. For g(x) = x and f(x) = 2x + 1, (gDf)(x) or g(f(x)) = 2x + 1Department of Mathematics, Sinclair Community College, Dayton, OH 2. If $f(x) = x_2 + 2$ and g(x) = 3 x-, then (fDg)(x) or f(g(x)) = ()2 3 x 2 + = 5 - x. For g(x) = 3x- and $f(x) = x_2 + 2$, (gDf)(x) or g(f(x)) =3 (x 2) + = 2 1 x Finding the domain of a composite function consists of two steps:

Step 1. Find the domain of the "inside" (input) function. If there are any restrictions on the domain, keep them.

Step 2. Construct the composite function. Find the domain of this new function. If there are restrictions on this domain, add them to the restrictions from Step 1. If there is an overlap, use the more restrictive domain (or the intersection of the domains). The composite may also result in a domain unrelated to the domains of the original functions. Examples:

1. In Example 2 above, the domain for g(x) = 3 x

- is $x \leq 3$.

The domain for f(g(x)) = 5 - x is all real numbers, but you must keep the domain of the inside function. So the domain for the composite function is also $x \le 3$.

2. Also in Example 2, the domain for $f(x) = x_2 + 2$ is all real numbers. The domain for the composite function g(f(x)) =

1 x

is $-1 \le x \le 1$. The input function f(x) has no restrictions, so the domain of g(f(x)) is determined only by the composite function.

3. Find fDg and g f and the domain of each, where f(x) = x D

 $_{2}^{2}+2$ and g(x) = 7 x

-.

f g: Step 1. What is the domain of the inside function g(x)? $x \le 7$ Keep this!

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D
Step 2. The composite f(g(x)) = (
)2
7 x
+2 = 7 - x + 2 = 9 - x
This function puts no additional restrictions on the domain, so the composite domain is x \le 7.
gDf: Step 1. What is the domain of the inside function f(x)?
All real numbers.
Step 2. The composite g(f(x)) =
7 (x
2)
5 x
-
+
=
This function creates new restrictions: the composite domain is -5 \le x \le 5.
4. Find fDg and g f and the domain of each, where f(x) =
D
1 x
3x
and g(x) =
1
1 3x
+
f g: Step 1. What is the domain of the inside function g(x)?
x \neq -1/3 Keep this!
D
Step 2. The composite f(g(x)) =
1
3x
1
1 3x
1 3x
3
1
3
1,3x
 3x
Ŋ
-|
|
|
}
+
₹
```

+

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ť
= \mathbf{x}
This function puts no additional restrictions on the domain, so the composite domain is x \neq -1/3.
gDf: Step 1. What is the domain of the inside function f(x)?
x \neq 0 Keep this!
Step 2. The composite g(f(x)) =
1
1
х
1
1 x
13
х
3x
=
=
Ŕ
+
l
This function puts no additional restrictions on the domain, so the composite domain is x \neq 0.
5. Find fDg and g f and the domain of each, where f(x) =
D
3x
x 1
and g(x) =
2
х
f g: Step 1. What is the domain of the inside function g(x)?
x \neq 0 Keep this!
D
Step 2. The composite f(g(x)) =
2
6
3
6
Х
х
2 x
2
2 x
1
х
=
=
1
```

Domain: $x \neq 2$ Combine this domain with the domain from Step 1: the composite domain is $x \neq 0$ and $x \neq 2$. Department of Mathematics, Sinclair Community College, Dayton, OH

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gDf: Step 1. What is the domain of the inside function f(x)?
x \neq 1 Keep this!
Step 2. The composite g(f(x)) =
2
2(x
3x
3x
x 1
1)
Domain: x \neq 0
Combine this domain with the domain from Step 1: the composite domain is x \neq 1 and x \neq 0.
Example of Intersection of Domains
6. Find fDg and g f and the domain of each, where f(x) =
D
x 2
and g(x) =
x 1
f g: Step 1. What is the domain of the inside function g(x)? x
D
_2 \ge 1 \rightarrow x \ge 1 \text{ or } x \le -1
Step 2. The composite f(g(x)) =
x 1 2
- - . The domain of this function is where
x 1
- \ge 2 \rightarrow x_2 - 1 \ge 4 \rightarrow x_2 \ge 5 \rightarrow x \ge 5 \text{ or } x \le -5
This function has a more restrictive domain than g(x), so the composite domain is
x \ge 5 or x \le -5. (This is the intersection of the two domains.)
gDf: Step 1. What is the domain of the inside function f(x)?
x \ge 2
Step 2. The composite g(f(x)) = (
)2
x 2
1
x 2 1
x 3
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-- = - - = - . Domain: $x \ge 3$ This function has a more restrictive domain than f(x), so the composite domain is $x \ge 3$. (This is the *intersection* of the two domains.) **Additional Problems** Find fDg, gDf, and the domain of each for the following functions. 1. f(x) = x + 3g(x) =9 x 4. $f(x) = x_2 + 2$ g(x) = x 52. f(x) = x 3+g(x) = 2x - 55. f(x) =2 x 3 g(x) = 5 x 2 $^+$ 3. f(x) =3 х g(x) = Х x 2 6. f(x) =1 x 2 $g(x) = x_2 - 3$ Department of Mathematics, Sinclair Community College, Dayton, OH 3

Answers

1. f(g(x)) =9 x - + 3 domain: -3 $\le x \le 3$ g(f(x)) =x 6x - domain: -6 $\le x \le 0$ 2. f(g(x)) = 2x 2domain: $x \ge 1$ Page 4

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g(f(x)) = 2 x 3
+ - 5
domain: x \ge -3
3. f(g(x)) =
3(x 2)
х
_
domain: x \neq 2 and x \neq 0
g(f(x)) =
3
3 2x
+
domain: x \neq 0 and x \neq -3/2
4. f(g(x)) = x - 3
domain: x \ge 5
g_{2}(f(x)) =
x 3
-
domain: x \ge 3 or x \le -3
5. f(g(x)) =
2(x 2)
3x 1
^+
-
+
domain: x \neq -2 and x \neq -1/3
g(f(x)) =
5(x 3)
2x 4
_
domain: x \neq 3 and x \neq 2
6. f(g(x)) =
3
2
1
x 3 2
- -
domain: x \neq
11
\pm
g(f(x)) =
(
)2
1
3
x 2
-
domain: x \neq 8
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