1. Write each equation in the form \( y = a(x - p)^2 + q \). Then:
   i) Sketch each parabola
   ii) Label the vertex, axis of symmetry and 2 other points on the graph
   iii) State the maximum or minimum value of \( y \).

   a) \( y = x^2 - 8x + 3 \)
   b) \( y = 2x^2 - 20x + 34 \)
   c) \( y = 3x^2 + 6x - 2 \)

   d) \( y = 4x^2 + 8x - 2 \)
   e) \( y = \frac{1}{2}x^2 + 6x + 15 \)
   f) \( y = \frac{1}{3}x^2 - \frac{4}{3}x + 2 \)

   g) \( y = -2x^2 + 4x \)
   h) \( y = -3x^2 - 12x - 13 \)
   i) \( y = -\frac{1}{2}x^2 - 2x + 2 \)

2. Two numbers have a difference of 6. Their product is a minimum. Determine the numbers.

3. The sum of two natural numbers is 20. Their product is a maximum. Determine the numbers.

4. The sum of two numbers is 28. The sum of their squares is a minimum. Determine the numbers.

5. Two numbers have a difference of 24. The result of adding their sum and their product is a minimum. Determine the numbers.

6. A lifeguard marks off a rectangular swimming area at the beach with 200m of rope. What is the greatest area of water she can enclose?

7. The daily profit of an ice cream stand is given by the function \( P = -30x^2 + 120x + 625 \) where \( P \) is the profit and \( x \) is the price of one ice cream cone (in dollars). What should the selling price of an ice cream cone be for maximum daily profit? What is the maximum daily profit?

8. A producer of synfuel from coal estimates that the cost \( C \) dollars per barrel for a production run of \( x \) thousand barrels is given by \( C = 9x^2 - 180x + 940 \). How many thousand barrels should be produced each run to keep the cost per barrel at a minimum? What is the minimum cost per barrel of synfuel?
Answers:

1a) \( y = (x - 4)^2 - 13 \)  
   Vertex: (4, -13)  
   Axis of symmetry: \( x = 4 \)  
   Min at \( y = -13 \)  

1b) \( y = 2(x - 5)^2 - 16 \)  
   Vertex: (5, -16)  
   Axis of symmetry: \( x = 5 \)  
   Min at \( y = -16 \)  

1c) \( y = 3(x + 1)^2 - 5 \)  
   Vertex: (-1, -5)  
   Axis of symmetry: \( x = -1 \)  
   Min at \( y = -5 \)  

1d) \( y = 4(x + 1)^2 - 6 \)  
   Vertex: (-1, -6)  
   Axis of symmetry: \( x = -1 \)  
   Min at \( y = -6 \)  

1e) \( y = \frac{1}{2}(x + 6)^2 - 3 \)  
   Vertex: (-6, -3)  
   Axis of symmetry: \( x = 6 \)  
   Min at \( y = -3 \)  

1f) \( y = \frac{1}{3}(x - 2)^2 + \frac{2}{3} \)  
   Vertex: \( \left( 2, \frac{2}{3} \right) \) 
   Axis of symmetry: \( x = 2 \)  
   Min at \( y = \frac{2}{3} \)  

1g) \( y = -2(x - 1)^2 + 2 \)  
   Vertex: (1, 2)  
   Axis of symmetry: \( x = 1 \)  
   Max at \( y = 2 \)  

1h) \( y = -3(x + 2)^2 - 1 \)  
   Vertex: (-2, -1)  
   Axis of symmetry: \( x = -2 \)  
   Max at \( y = -1 \)  

1i) \( y = -\frac{1}{2}(x + 2)^2 + 4 \)  
   Vertex: (-2, 4)  
   Axis of symmetry: \( x = -2 \)  
   Max at \( y = 4 \)  

2. 3 & -3  

3. 10 and 10  

4. 14 and 14  

5. 11 and -13  

6. 5000m^2  

7. Selling price = $2, maximum profit = $745  

8. Produce 10 thousand, minimum cost = $40/barrel